with the barometric maximum in 10 districts, and follows it by about one month in 9, and by about two months in 4, districts; the other cases generally admitting of some explanation.

- 3. In several insular seismic districts, and especially in Japan and New Zealand, the amplitude of the annual period is very small; and, if many of the earthquakes of these districts originate beneath the sea, this should be the case; for, in the course of a year, as the barometric pressure changes, the sea will have time to take up its equilibrium position, and thus the total pressure on the sea-bottom will be unaltered.
- V. "Electrical Interference Phenomena somewhat Analogous to Newton's Rings, but exhibited by Waves passing along Wires of which a part differs from the rest." By EDWIN H. BARTON, B.Sc., "1851 Exhibition" Science Scholar. Communicated by Professor A. W. RUCKER, M.A., F.R.S. Received May 25, 1893.
- 1. In 1891 Mr. V. Bjerknes* showed how to measure the wavelength and primary damping of the electrical oscillations in a Hertzian primary conductor by the use of a special electrometer and long parallel wires along which induced oscillations were propagated. This form of Hertzian secondary conductor, in which the wires are far too long to be in resonance with the primary oscillator, may, hereafter in this paper, be referred to as "the long secondary" or simply "the secondary."
- 2. During the following session Herr von Geitler† found that, if the wires at any part of the long secondary were either
 - (1) Replaced by others thicker or thinner than the normal wires, or
 - (2) Arranged nearer together or further apart than the normal distances,

then in any of these cases, a partial reflection of the electrical waves occurred at such place of change in the wires.

Herr von Geitler then made further observations of what occurred when a condenser was attached at a *single point* of each wire, but did not quantitatively examine the effect produced on the waves by a finite length of the secondary being different from the rest.

- * 'Wiedemann's Annalen,' vol. 44, pp. 513—526, 1891, "Ueber den Zeitlichen Verlauf der Schwingungen im primären Hertz'schen Leiter," von V. Bjerknes aus Christiania.
- † 'Wiedemann's Annalen,' vol. 49, pp. 184—195, 1893, "Ueber Reflexion electrischer Drahtwellen," von J. Ritter von Geitler.

- 3. Following the researches of these physicists, and in the same laboratory,* I have endeavoured to trace, both theoretically and experimentally, the relation between the length of this abnormal part of the "secondary" and the relative intensities of the transmitted and reflected disturbances into which the original incident wave is thereby divided.
- 4. The following diagram and accompanying descriptive notes will sufficiently explain the apparatus used in the experiments:—

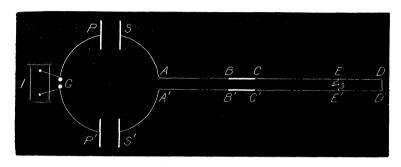


Fig. 1.—Diagrammatic Outline of the Apparatus used for producing and measuring the Interference of Electrical Oscillations.

EXPLANATION OF FIG. 1.

I. Induction coil worked by two secondary cells.

G. Spark gap (usually 2 mm.).

PGP'. (Measured along the wires) is 204 cm., the wires PG, GP' are 2 mm. diameter.

PP'. Condenser plates of zinc 40 cm. diam. to form the ends of the Hertzian primary oscillator.

SS'. Similar plates at a distance of 30 cm. from P and P', and forming the beginning of the "long secondary," which consists of copper wires 1 mm. diameter.

Distance AA' = BB' = CC' = DD' = 8 cm.

BC B'C'. The abnormal part of the long secondary used to produce the reflection and interference phenomena.

EE'. Electrometer.† The needle is uncharged: it therefore turns in the same direction whenever there is any potential difference between E and E', whatever the sign of that difference.

DD'. Wire bridge across the main wires.

Length AD = A'D' = 160 m, nearly. ED = $\frac{1}{4}\lambda_1$ where λ_1 denotes the wave-length in the long secondary.

^{*} Namely, in the University of Bonn, under the guidance of Professor Hertz.

^{.†} Herr von Geitler kindly left for my use the instrument made by him and described in 'Wiedemann's Annalen,' vol. 49, p. 188. I am also indebted to him for full verbal explanations of his researches prior to their publication, and hereby tender him my hearty thanks.

Divisions of Theory.

5. Imagine an electrical conductor ABCD (see fig. 1) consisting of three parts, AB, BC, and CD, in each of which parts the electrostatic capacity and other properties of the conductor remain constant, those of the third part CD being precisely like those of the first part AB, but the second part BC differing from the other two parts either in its own dimensions or in the nature of the dielectric* by which it is surrounded, or in both these respects.

And consider an electrical wave passing along this conductor from A towards D. Suppose its amplitude in the part AB is a, and that from the point B (immediately after the instant of incidence of the wave there and before any disturbance has reached C), a wave of amplitude ab is reflected towards A, and a wave of amplitude ac transmitted within the part BC towards C. The constants b and c may be referred to as the coefficients of reflection and transmission respectively.

- 6. Then the elementary mathematical investigation of what may be expected to occur falls naturally into three parts, namely:—
- (1) The derivation of the coefficients of reflection and transmission at the point B from the changes in the properties of the conductor which occur there.
- (2) The relation between these coefficients and the similar ones involved when the wave reaches C and thus encounters the reverse change in the properties of the conductor.
- (3) The determination of the intensities of the total disturbances reflected at or passing through B in the direction A and of those transmitted through C in the direction D respectively, each being the result of an infinite series of interfering waves produced by multiple reflections within the part BC, these again being produced by the original wave passing along AB.

These three branches of the theory will now be taken in the above order.

- 7. I. Theory of single reflection and transmission of an electrical wave along a conductor at a point where either its electrostatic capacity, or its coefficient of self-induction, or both, change abruptly.—Let the following symbols be used, the same system of units being understood throughout:—
 - φ . Electrostatic potential.
 - C. Electrostatic capacity per unit length of conductor.
- * It is to be distinctly understood throughout that the medium surrounding all parts of the conductor is supposed to be a dielectric. All idea of its possessing appreciable conductivity, and consequently absorbing a sensible portion of the energy of the wave, is excluded from this theory. Also for all parts of the conductor itself let w (the magnetic permeability) = unity.

- Q. Quantity of electricity per unit length of conductor.
- L. Coefficient of self-induction ,,
- R. Resistance "
- i. Electric current.
- v. Velocity of propagation of the waves along the conductor.
- λ. Wave-length.
- t. Time.
- L, R, and v denote the values corresponding to the high frequencies used.

Take the conductor as the axis of x.

For the normal parts of the conductor, namely, AB and CD, fig. 1, the above symbols will be used with the subscript 1; for the abnormal part BC they will be used with the subscript 2.

8. When an electrical wave passes along a conductor we have at any point the E.M.F. $=-\frac{\partial \phi}{\partial x}-L\frac{\partial i}{\partial t}$. But this also equals Ri. Thus, since $i=Qv=C\phi v$, we obtain the differential equation

$$\frac{\partial \phi}{\partial x} + v CL \frac{\partial \phi}{\partial t} + v CR \phi = 0 \quad ... \quad (1).$$

When and where $\phi=0$, we have

$$\frac{\partial \phi}{\partial x} + v \text{CL} \frac{\partial \phi}{\partial t} = 0 \quad \text{or} \quad \frac{\partial \phi}{\partial t} \left[v \text{CL} - \frac{1}{v} \right] = 0,$$

$$v = \frac{1}{\sqrt{\text{CL}}} \quad \dots \qquad (2),$$

whence

the well known expression for the velocity of propagation of the wave.

9. Now R in equation (1) leads to a damping factor in the solution. Since, however, we are now concerned simply with what occurs at the point of reflection, this R will be omitted. Equation (1) then becomes

$$v\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial t} = 0$$

$$\phi = f_1(\beta t - \beta_0 x) + f_2(\beta t + \beta_0 x)$$

$$(3)$$

and its solution

where $\beta/\beta_0 = v = 1/\sqrt{\text{(CL)}}$, and f_1, f_2 denote any functions.

10. It will suffice for the case in question if we write for f_1 and f_2 sine functions with coefficients for the various amplitudes, and a third term in the brackets to allow for a change of phase should there be one. We may thus write for the original and reflected waves in the first part of the conductor

$$\phi_1 = a \sin (\beta t - \beta_1 x) + ab \sin (\beta t + \beta_1 x + \delta_1) \dots (4)$$

And for the transmitted wave in the second part of the conductor

where

Further, remembering that $i = Q \times (\pm v) = C\phi \times (\pm v)$, we have from (4) and (5)

$$i_1 = C_1 v_1 a \sin (\beta t - \beta_1 x) - C_1 v_1 ab \sin (\beta t + \beta_1 x + \delta_1) \dots$$
 (6),

and
$$i_2 = C_2 v_2 ac \sin (\beta t - \beta_2 x + \delta_2)$$
 (7).

11. Now take B, the junction of the two parts of the conductor, as the origin of abscissæ. Then, for x = 0, we have $\phi_1 = \phi_2$ and $i_1 = i_2$; unless $\partial \phi / \partial x$ and $\partial i / \partial x$ become infinite at that point. Applying this relation to equations (4), (5), (6), and (7) yields

$$\sin \beta t + b \sin (\beta t + \delta_1) = c \sin (\beta t + \delta_2) \dots (8).$$

$$C_1v_1\lceil\sin\beta t - b\sin(\beta t + \delta_1)\rceil = C_2v_2c\sin(\beta t + \delta_2)\dots (9).$$

12. Now equations (8) and (9) hold good for all values of t; hence in each of them we may equate the coefficients of $\sin \beta t$ and of $\cos \beta t$ respectively after expanding the sines. This leads to four equations from which to determine the four unknowns. The solution may be written as follows:—

$$\delta_{1} = 0, \qquad \delta_{2} = 0,
b = \frac{C_{1}v_{1} - C_{2}v_{2}}{C_{1}v_{1} + C_{2}v_{2}} = \frac{\sqrt{(C_{1}/L_{1}) - \sqrt{(C_{2}/L_{2})}}}{\sqrt{(C_{1}/L_{1}) + \sqrt{(C_{2}/L_{2})}}} \right\} \dots (10).$$

$$c = \frac{2 C_{1}v_{1}}{C_{1}v_{1} + C_{2}v_{2}} = \frac{2 \sqrt{(C_{1}/L_{1})}}{\sqrt{(C_{1}/L_{1}) + \sqrt{(C_{2}/L_{2})}}}$$

- 13. The following results of (10) may be noted:-
- (1) Energy of the original wave is proportional to a constant $\times C_1v_1a^2$, that of the reflected wave to a constant $\times C_1v_2a^2b^2$, and that of the transmitted wave to a constant $\times C_2v_2a^2c^2$. We ought, therefore, to have—

$$C_1v_1 = \mathbf{C}_1v_1b^2 + C_2v_2c^2 \dots (11)$$

And this equation is satisfied by the values of b and c in (10).

(2) If $L_2 = L_1$ we have—

$$b = \frac{\sqrt{C_1 - \sqrt{C_2}}}{\sqrt{C_1 + \sqrt{C_2}}} = -\frac{v_1/v_2 - 1}{v_1/v_2 + 1}$$

$$c = \frac{2}{\sqrt{C_1 + \sqrt{C_2}}} = \frac{2}{v_1/v_2 + 1}$$
(12).

and

[Compare Preston's 'Theory of Light,' 1890, pp. 285—286.]

(3) If $C_2L_2 = C_1L_1$, then $v_2 = v_1$, $\lambda_2 = \lambda_1$, and we have

$$b = \frac{C_1 - C_2}{C_1 + C_2} = -\frac{C_2/C_1 - 1}{C_2/C_1 + 1}$$

$$c = \frac{2C_1}{C_1 + C_2} = \frac{2}{C_2/C_1 + 1}$$
(13).

and

II. Relation between the various coefficients involved when the second part of the conductor has finite length, and is succeeded by a third part like the first.—This abnormal intermediate part of the conductor may sometimes be referred to as "the condenser."

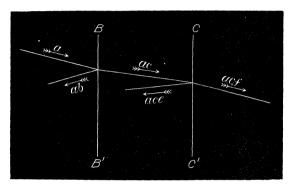


Fig. 2.—Diagrammatic View of Waves, drawn, for clearness' sake, as though they were rays incident and reflected obliquely. BB' shows the boundary between the first and second parts of the conductor; CC' shows that between the second and third parts. The small letters near the arrows are the amplitudes of the waves.

The values of b and c in fig. 2 have already been determined, and we have by the same equation [viz.: (10)]

$$e = \frac{C_2 v_2 - C_1 v_1}{C_2 v_2 + C_1 v_1} = -b,$$
and
$$f = \frac{2 C_2 v_2}{C_2 v_2 + C_1 v_1},$$
whence
$$cf = \frac{4 C_1 v_1 C_2 v_2}{(C_1 v_1 + C_2 v_2)^2} = 1 - b^2.$$

III. Theory of the multiple internal reflections of a damped electrical wave in the abnormal part of a conductor (the previous and succeeding parts being alike) with expressions for the intensities of the resulting transmitted and reflected disturbances.—Let the equation of the wave in the first part of the conductor be

$$y = \alpha e^{-\alpha t + \alpha_1 x} \sin(\beta t - \beta_1 x) \dots (15),$$

$$\alpha / \alpha_1 = \beta / \beta_1 = v_1.$$

where

(Similarly $\alpha/\alpha_2 = \beta/\beta_2$ will be used for the value v_2 .)

It will thus be seen that the damping just referred to in the heading of this paragraph is the primary damping, that is, the time-rate of decrease of the oscillations occurring in the Hertzian primary conductor. Secondary damping, on the other hand, refers to the space-rate of decrease of any individual wave as it proceeds along the Hertzian secondary conductor. It is, of course, along the long form of this secondary conductor that we are now supposing the waves to travel, but the secondary damping is known to be small in comparison with the primary damping, and is, therefore, in the present part of the theory, legitimately neglected.*

16. It will readily be seen that the ordinary mathematical treatment of the interference of light in thin plates will not strictly apply to this case.

For, in the optical phenomenon, one supposes a continuous beam of light of constant amplitude. We may, therefore, in that case, at once take, to *infinity*, the sums of the series of reflected and transmitted rays to which the original one gives rise, and *neglect* the comparatively *small period which elapses before those two infinite series are made up*, and during which (the series being as yet incomplete) the reflected and transmitted beams have not reached their final steady values.

But with such primary damping as that with which we have to do (namely, of the order $\gamma_1 = 2\pi\alpha/\beta = 0.5$), the character of the result would be essentially changed by the unwarrantable assumption that the amplitude of the incident wave remains sensibly constant until the infinite series of internal reflections has taken place.

17. The question is therefore attacked by forming a series of integrals.

Referring to fig. 1 or fig. 2, the wave given by equation (15) advances in the positive direction along the axis of x, that is, in the direction ABCD. Let t=0 when the head of the wave first reaches C. And let x=0 for the wave which is at the point C, without having suffered any internal reflections in the part BC. Thus, for a wave which has suffered 2n internal reflections within the part BC, the point C has for its abscissa $2n \times l$, where l denotes the length BC.

Now let y_n denote a wave emerging at C after 2n internal reflections in BC; then we have, by putting, in equation (15), x = 2nl, and supplying the amplitude from (14),

$$y_{n} = ab^{2n} (1-b^{2}) e^{-at+a_{2}^{2}nl} \sin (\beta t - \beta_{2}^{2} 2nl)$$
or
$$y_{n} = ab^{2n} (1-b^{2}) e^{-a(t-nt_{2})} \sin [\beta (t-nt_{2})],$$
where
$$t_{2} = 2l/v_{2}$$
(16)

^{*} Compare 'Wiedemann's Annalen,' vol. 44, pp. 83 and 515, 1891.

It must now be recollected that, since the electrometer needle is uncharged, it takes no account of the sign of the potential difference between E and E', fig. 1, but gives a deflection proportional to $y^2 dt$, taken between the proper time limits.

18. Hence, if E denotes the electrometer constant, and Io, It, and Ir, its deflections for the passage of the wave-train, without the condenser, after transmission through the condenser (as shown in fig. 1), and by reflection from the condenser, respectively, we have

$$EI_{0} = \int_{0}^{\infty} y_{0}^{2} dt$$
and
$$EI_{t} = \int_{0}^{t_{2}} y_{0}^{2} dt + \int_{t_{2}}^{2t_{2}} (y_{0} + y_{1})^{2} dt + \int_{2t_{2}}^{3t_{2}} (y_{0} + y_{1} + y_{2})^{2} dt$$

$$+ \dots + \int_{nt_{2}}^{(n+1)t_{2}} (y_{0} + y_{1} + y_{2} + \dots + y_{n})^{2} dt$$

$$+ \dots \quad ad \quad inf. \tag{17}$$

And in these equations everything is expressible in terms of quantities considered known.

19. The evaluation of the second part of (17) (being a doublyinfinite series of definite integrals) is a somewhat long process, but rigidly performed to infinity, and the result divided by that for EI to eliminate E, and the like operation for I_r , we have

$$I_{t}/I_{0} = \frac{1-b^{2}}{1+b^{2}} + U,$$

$$I_{r}/I_{0} = \frac{2b^{2}}{1+b^{2}} - U,$$
where
$$U = \frac{1-b^{2}}{1+b^{2}} \cdot \frac{2b^{2}e^{-at_{2}}}{\beta} \cdot \frac{(\alpha \sin \beta t_{2} + \beta \cos \beta t_{2} - \beta b^{2}e^{-at_{2}})}{1-2b^{2}e^{-at_{2}}\cos \beta t_{2} + b^{4}e^{-2at_{2}}}$$
(18)

where

- 20. The following results of equation (18) may be noticed.
- (1) $I_t/I_0 + I_r/I_0 = 1$ or $I_t + I_r = I_0$ for all values of b and t_2 , as should be the case.
- (2) On putting $t_2 = 0$ or b = 0, that is, removing the condenser, we have

$$I_t/I_0 = 1$$
 or $I_t = I_0$, all transmitted,

and

$$I_r = 0$$
, none reflected.

On the other hand, with b = 1, we get all reflected and none transmitted, unless $t_2 = 0$.

(3) On differentiating U to t_2 , and putting $\partial U/\partial t_2 = 0$ to obtain the values of l which give the stationary values of I_t and I_r , we obtain $\sin \beta t_2 = 0$, whence $l = \frac{1}{4}(n\lambda_2)$, where n is any integer.

min.

max.

 λ_2 . $2n\frac{\lambda_2}{4}$. $(2n+1)\frac{\lambda_2}{4}$. Values of l. 0. Nature of value of I_t $\max = I_0$. max. max. max.

min.

max.

min.

min.

The values occur as in the following table:-

min.

max.

(4) We see, on inspection of (18), that the values of I_t and I_r (when plotted as the ordinates, the values of l being the abscissæ) form a damped wavy curve, but that neither the damping nor the wave-form are of the simplest type.

min.

(5) On putting $t_2 = \infty$, U disappears, and we have

$$I_t/I_0 = \frac{1-b^2}{1+b^2}, \qquad I_r/I_0 = \frac{2b^2}{1+b^2} \dots (19).$$

(6)* On putting $\alpha = 0$, that is, removing the primary damping from the expression for the wave, we obtain

$$I_{t}/I_{0} = \frac{(1-b^{2})^{2}}{1-2b^{2}\cos\beta t_{2}+b^{4}}$$

$$I_{r}/I_{0} = \frac{4b^{2}\sin^{2}(\beta t_{2}/2)}{1-2b^{2}\cos\beta t_{2}+b^{4}}$$
(20)

and

Nature of value of I_r .

the ordinary expressions for the case of the interference of light in thin plates (see, e.g., Preston's 'Theory of Light,' 1890, pp. 145-147).

21. The theoretical values of I_t/I_0 from equation (18) are plotted in curve No. 1, for the following values of the constants, the abscissæ representing l, and the ordinates I_l/I_0 .

log. dec.
$$\gamma_1 = 2\pi \alpha/\beta = 0.5$$
, $C_2L_2 = C_1L_1$; $\lambda_2 = \lambda_1 \text{ and } v_2 = v_1$, $\lambda_1 = 9 \text{ m.}$, $C_2 = 9 C_1$, $b = 0.8$.

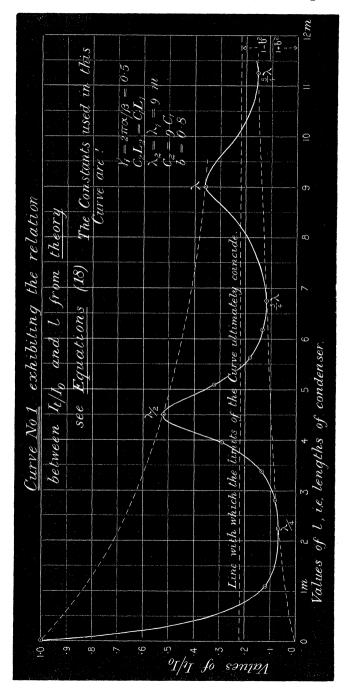
therefore

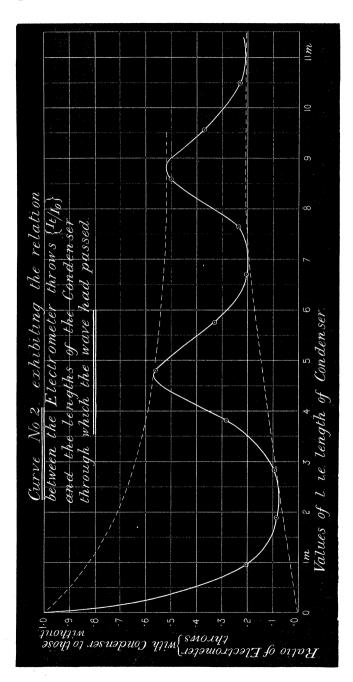
whence

by equation (13).

^{*} For this suggestion and for much valuable help in checking mathematical working and results I am indebted to Mr. G. Udny Yule.







- 22. The Experiment.—In the experiments performed hitherto, I have made the abnormal part of the conductor by hanging upon the wires of the long secondary sheets of tinfoil 32 cm. deep, the length varying up to 105 m. Several observations of the electrometer throws are taken without the condenser, several with the condenser 1 m. long, several with the condenser 2 m. long, and so forth; check observations being taken while the condenser is being shortened again. Curve No. 2 is plotted with condenser lengths as abscissæ, and electrometer throws as ordinates; these latter, however, being reduced to the scale, electrometer throws without condenser equals unity. They thus compare with the values of I_t/I_0 in curve No. 1. The wave-length used was $\lambda_1 = 9$ m.
- 23. It is seen that the experimental curve thus obtained agrees in its general form with that plotted from theoretical considerations. Exact coincidence of theory and experiment cannot at this stage be expected. I have, accordingly, made no attempt to plot a curve from equation (18) with values of the constants which profess to exactly represent those involved in the experiment.
- 24. I am aware of two chief sources of disturbances in the experimental conditions, but have already shown that they are not of such order as to invalidate the above results, which, therefore, hold good as first approximations.
- 25. As the present paper is only a preliminary one, intended to give an outline of the theory and experiment, I will not now enlarge upon the topic of disturbances. I am still engaged on these interference phenomena, and hope to publish a full account of the results next session.

In conclusion, I wish to express my great indebtedness to Professor Hertz, both for first directing my attention to the subject of these reflections, and also for his invaluable advice in the course of the work.

VI. "On Interference Phenomena in Electric Waves passing through different Thicknesses of Electrolyte." By G. UDNY YULE. Communicated by Professor G. CAREY FOSTER, F.R.S. Received May 31, 1893.

In the spring of 1889 Professor J. J. Thomson published* a description of some experiments made by him for comparing the resistances of electrolytes to the passage of very rapidly alternating currents, the method consisting in comparing the thicknesses of layers of different electrolytes which were equally opaque to Hertzian radiation. During

^{* &#}x27;Roy. Soc. Proc.,' vol. 45, p. 269, 1889.

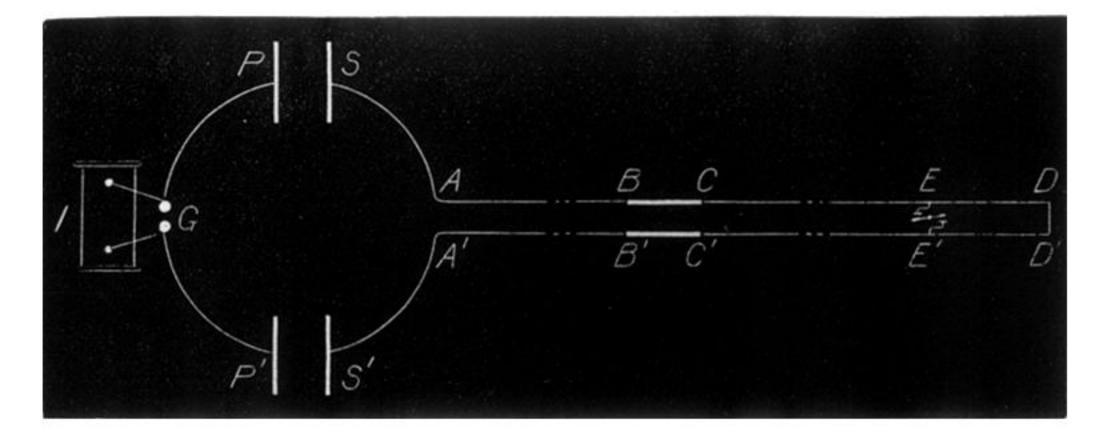


Fig. 1.—Diagrammatic Outline of the Apparatus used for producing and measuring the Interference of Electrical Oscillations.

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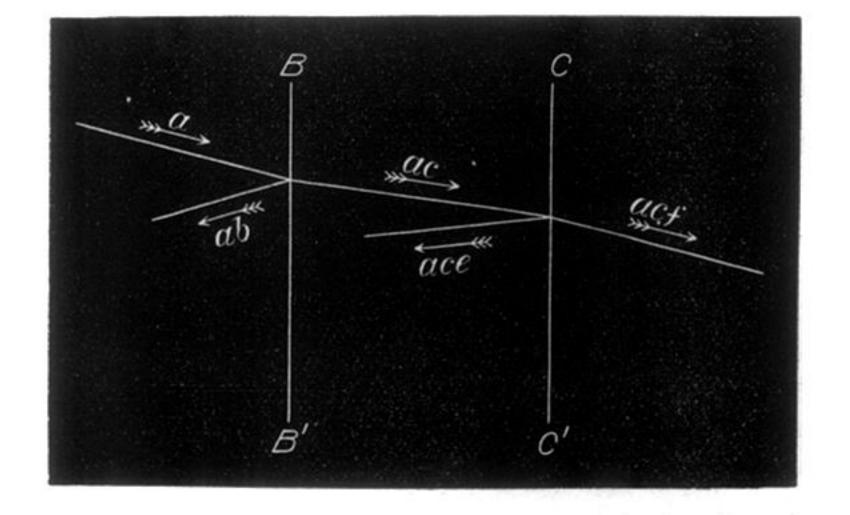


Fig. 2.—Diagrammatic View of Waves, drawn, for clearness' sake, as though they were rays incident and reflected obliquely. BB' shows the boundary between the first and second parts of the conductor; CC' shows that between the second and third parts. The small letters near the arrows are the amplitudes of the waves.

